

BOUNDARY LAYER FLOW AND HEAT
TRANSFER FOR MAXWELL FLUID AND
VISCOELASTIC NANOFUID OVER A
STRETCHING SHEET

NAZILA BT ISHAK

MASTER OF SCIENCE
(MATHEMATICS)

UNIVERSITI MALAYSIA PAHANG



SUPERVISOR'S DECLARATION

We hereby declare that We have checked this thesis and in our opinion, this thesis is adequate in terms of scope and quality for the award of the degree of Master of Science (Mathematics).

(Supervisor's Signature)

Full Name : DR. MOHD ZUKI BIN SALLEH

Position : PROFESSOR

Date :

(Co-supervisor's Signature)

Full Name : DR. NORHAYATI BINTI ROSLI

Position : ASSOCIATE PROFESSOR

Date :



STUDENT'S DECLARATION

I hereby declare that the work in this thesis is based on my original work except for quotations and citations which have been duly acknowledged. I also declare that it has not been previously or concurrently submitted for any other degree at Universiti Malaysia Pahang or any other institutions.

(Student's Signature)

Full Name : NAZILA BT ISHAK

ID Number : MSE 12001

Date :

BOUNDARY LAYER FLOW AND HEAT TRANSFER FOR MAXWELL FLUID
AND VISCOELASTIC NANOFLUID OVER A STRETCHING SHEET

NAZILA BT ISHAK

Thesis submitted in fulfillment of the requirements
for the award of the degree of
Master of Science
(Mathematics)

Centre For Mathematical Sciences
UNIVERSITI MALAYSIA PAHANG

NOVEMBER 2019

ACKNOWLEDGEMENTS

Special praise to Allah, first because has given good health to complete this work.

Thanks to my supervisor, Prof. Dr Mohd Zuki Salleh for his valuable guidance, endless encouragement, sharing the knowledge, financial support and always assisting decision making in the study for the success of this study. I really appreciate his contributions from the first day enrolling as a student.

I would also like to thank my co-supervisor, Assoc. Prof. Dr. Norhayati Binti Rosli on valuable assistance and suggestions. Not forgetting thanks to Muhammad Khairul Anuar Mohamed for his support and mentoring. Without their efforts, this thesis will not come true in the end. In addition, I would also like to thank the staffs of the Faculty of Industrial Sciences & Technology (FIST), UMP who helped me in various ways to facilitate any matters related to the administration.

With sincere I would also like to express my thanks to my husband, my kids, my parents, my siblings, my relatives and my friends. Thanks to them for their ongoing love, support, care, help and time sacrifice during the period of this study. There are no enough words to describe all my appreciation for them.

Lastly, thanks a lot to my committee members for their comments and suggestions, which led me to the successful completion of this study. Also, I would like to thank Universiti Malaysia Pahang (RDU121302 and RDU170358) and Ministry of Higher Education (RDU140111 and RDU150101) for financial supports.

ABSTRAK

Permasalahan aliran lapisan sempadan banyak diaplikasikan dalam industri dan kejuruteraan. Aplikasi ini adalah pelakaran filem plastik, penghasilan gentian kaca, penggelekan panas dan pelbagai proses dalam industri pembuatan. Ciri-ciri produk akhir bergantung kepada cecair pendingin yang digunakan dan kadar regangan. Bendalir tak-Newtonan boleh dibahagikan kepada beberapa kategori seperti viscoelastik, kelikatan kebergantungan-masa dan kelikatan tak-Newtonan. Contoh bendalir ini adalah seperti minyak, sos, pes makanan, cat dan larutan koloid. Kajian bendalir tak-Newtonan mendapat lebih perhatian kerana prestasinya yang baik di dalam industri dan penggunaan teknologi jika dibandingkan dengan bendalir Newtonan. Dalam tesis ini, terdapat dua jenis bendalir tak-Newtonan yang dicadang dan diperiksa iaitu cecair Maxwell dan nanobendalir visco elastik. Terdapat dua jenis syarat sempadan yang telah dikaji dalam tesis ini iaitu syarat sempadan yang ditetapkan suhu dinding dan keadaan gelinciran. Model yang dicadangkan untuk setiap permasalahan bergantung kepada sistem persamaan menakluk yang selari dengan syarat awal dan syarat sempadan. Pembolehubah tak berdimensi yang sesuai diperkenalkan dan diperturunkan kepada bentuk persamaan menakluk tak berdimensi. Penyelesaian berangka bagi persamaan pembezaan biasa diselesaikan dengan menggunakan kaedah Luruan. Seterusnya, kaedah Luruan dijalankan untuk menyelesaikan hasil persamaan-persamaan pembezaan biasa menggunakan atur cara sedia ada dalam perisian MAPLE-13. Penyelesaian ini perlu asimptot dan memuaskan syarat awal dan syarat sempadan. Pengiraan berangka dilakukan untuk pelbagai nilai parameter tak berdimensi seperti nombor Prandtl Pr , parameter regangan ε , parameter sedutan S , parameter magnetik M , parameter Maxwell β , parameter Brown N_b , parameter thermopil N_t , parameter sinaran terma N_R , nombor Lewis Le , parameter gelinciran halaju dan terma dipertimbangkan. Perbandingan keputusan sekarang dengan hasil yang telah diterbitkan terdahulu telah dilakukan dan didapati sangat memuaskan. Keputusan berangka yang dipersembahkan dalam kajian ini adalah profil suhu, profil halaju, pekali geseran kulit, pekali pemindahan haba, nombor Nusselt dan nombor Sherwood. Peningkatan parameter regangan dan ciri-ciri terma menghasilkan penurunan pada suhu dinding dan ketebalan terma lapisan sempadan. Selain itu, peningkatan pada parameter Maxwell dan sedutan membawa kepada peningkatan pemindahan haba setempat manakala penurunan terhadap pekali geseran kulit, suhu dan halaju profil. Kehadiran parameter gelinciran halaju telah mempercepatkan pergerakan zarah-zarah bendalir dari keadaan pegun kepada halaju tertentu di permukaan. Keadaan ini telah mengurangkan pekali geseran kulit dan juga kesan parameter regangan.

ABSTRACT

The problem of boundary layer flow has many applications in industry and engineering field. Some of these applications are drawing of plastic films, glass fiber production, hot rolling and many others in industrial manufacturing processes. The final product requested characteristics depends on the cooling liquid used and the rate of stretching. Non-Newtonian fluids can be defined in several categories like viscoelastic, time-dependent viscosity and non-Newtonian viscosity. Such fluids like oils, ketchup, food paste, paints and colloidal solutions. The study of non-Newtonian fluids has gain great attraction due to their better performance in industrial and technological applications if compared to Newtonian fluids. In this thesis, there are two types of non-Newtonian fluids namely Maxwell fluids and viscoelastic nanofluids has been examined and proposed. There are two types of boundary conditions that will be studied in this thesis, that are prescribed wall temperature (PWT) and slip condition. The proposed model for each problem depends on the system of governing equations which along with imposed initial and boundary conditions. Suitable non-dimensional variables are then introduced and reduce the governing equations into dimensionless form. The numerical solutions of ordinary differential equations are solved by shooting method. Then, shooting method is carried out to solve the resulting system of ordinary differential equations through "build in" program in MAPLE-13 software. These solutions satisfy and asymptotic for the applied of imposed initial and boundary conditions. Numerical computations are carried out for various values of the parameters of the problems, which include the Prandtl number Pr , stretching parameters ε , suction parameters S , magnetic parameter M , Maxwell parameters β , Brownian parameter N_b , thermophoresis parameter N_t , thermal radiation parameters N_R , Lewis number Le , velocity and thermal slip parameter are considered. Comparisons of present result with previously published results are done and it is found to be in a good agreement. Numerical results presented in this study are the temperature profile, the velocity profile, the skin friction coefficient, the local heat transfer coefficient, the Nusselt number and the Sherwood number. The rises of stretching parameter and thermal characteristics are resulting on decreasing in the wall temperature and thermal boundary layer thickness. Other than that, the increasing in the Maxwell and suction parameter leads to increase on the local heat transfer while the skin friction coefficient, velocity and temperature profile become decrease. The presence of velocity slip parameter has accelerated the movement of fluid particle at surface from static to a certain velocity. This situation reduces the skin friction coefficient as well as the effect on stretching parameters.

TABLE OF CONTENT

DECLARATION	
TITLE PAGE	
ACKNOWLEDGEMENTS	ii
ABSTRAK	iii
ABSTRACT	iv
TABLE OF CONTENT	v
LIST OF TABLES	viii
LIST OF FIGURES	x
LIST OF SYMBOLS	xiii
LIST OF ABBREVIATIONS	xvi
CHAPTER 1 INTRODUCTION	1
1.1 Introduction	1
1.2 Research Background	2
1.2.1 Boundary Layer Theory	3
1.2.2 Stretching Surface	5
1.2.3 Magnetohydrodynamic (MHD) Flow	6
1.2.4 Maxwell Fluid	7
1.2.5 Nanofluids	8
1.3 Problem Statement	10
1.4 Objectives and Scope of Research	10
1.5 Research Methodology	11
1.6 Significance of the Study	12

1.7	Literature Review	12
1.8	Thesis Outlines	18
CHAPTER 2 GOVERNING EQUATION AND NUMERICAL METHOD		20
2.1	Introduction	20
2.2	Governing Equations	22
2.3	Similarity Transformations	24
	2.3.1 Continuity Equation	26
	2.3.2 Momentum Equation	27
	2.3.3 Energy Equation	29
	2.3.4 Derivation of Boundary Conditions	31
2.4	Numerical Method: Shooting Method	33
2.5	Methodology Flow Chart	35
CHAPTER 3 MHD FLOW AND HEAT TRANSFER FOR MAXWELL FLUID OVER A STRETCHING SHEET		37
3.1	Introduction	37
3.2	Mathematical Formulation	38
3.3	Results and Discussions	40
3.4	Conclusion	50
CHAPTER 4 THERMAL RADIATION EFFECTS ON STAGNATION POINT FLOW PAST A STRETCHING SHEET FOR MAXWELL FLUID WITH SLIP CONDITION		51
4.1	Introduction	51
4.2	Mathematical Formulation	52
4.3	Results and Discussions	56

4.4	Conclusion	65
CHAPTER 5 HEAT AND MASS TRANSFER FLOW OF A VISCOELASTIC NANOFLUID OVER A STRETCHING SHEET WITH SLIP CONDITION		66
5.1	Introduction	66
5.2	Mathematical Formulation	67
5.3	Results and Discussions	71
5.4	Conclusion	79
CHAPTER 6 CONCLUSION		80
6.1	Conclusion of Research	80
6.2	Contribution of the Research	81
6.3	Recommendation for Future Study	82
REFERENCES		83
APPENDIX A		95
APPENDIX B		98
APPENDIX C		109
APPENDIX D		119

LIST OF TABLES

Table 3.1	Comparison of the skin friction coefficient $f''(0)$ for different values of Maxwell parameter β in the case of stretching sheet $\varepsilon = 1$	40
Table 3.2	Values of the skin friction coefficient $f''(0)$ and local heat transfer coefficient $-\theta'(0)$ for different values of suction parameter S	41
Table 3.3	Values of the skin friction coefficient $f''(0)$ and local heat transfer coefficient $-\theta'(0)$ for different values of the Maxwell parameter β	41
Table 3.4	Values of the skin friction coefficient $f''(0)$ and local heat transfer coefficient $-\theta'(0)$ for different values of the stretching parameter ε	42
Table 3.5	Values of the skin friction coefficient $f''(0)$ and local heat transfer coefficient $-\theta'(0)$ for different values of the magnetic Parameter M	42
Table 4.1	Comparison of the skin friction coefficient $f''(0)$ for a different values of Maxwell parameter β	57
Table 4.2	Comparison of the wall temperature $\theta(0)$ for different values of Prandtl number Pr	57
Table 4.3	Values of the $\theta(0)$ and $-\theta'(0)$ for different values of the thermal radiation parameter N_R	58
Table 4.4	Values of the skin friction coefficient $f''(0)$ and wall temperature $\theta(0)$ for different values of the Maxwell parameter β	58
Table 4.5	Values of the skin friction coefficient $f''(0)$ and wall temperature $\theta(0)$ for a different values of dimensionless velocity slip parameter γ	59
Table 4.6	Values of the skin friction coefficient $f''(0)$ and wall temperature $\theta(0)$ for different values of stretching parameter ε	59

Table 5.1	Comparison between the present results of $f''(0)$ for different values of Prandtl parameter ε	72
Table 5.2	Comparison between the present results of local heat transfer $-\theta'(0)$ for different values of Prandtl parameter Pr	72
Table 5.3	Values of $Nu_x \text{Re}_x^{-1/2}$ and $Sh_x \text{Re}_x^{-1/2}$ for various values of ε and K when $N_b = N_t = 0.1$, $Le = 15$, $\gamma = 0.5$ and $\text{Pr} = 7$	73
Table 5.4	Values of $Nu_x \text{Re}_x^{-1/2}$ and $Sh_x \text{Re}_x^{-1/2}$ for various values of N_b , N_t and K when $Le = 15$, $\varepsilon = \gamma = 0.5$ and $\text{Pr} = 7$	73

LIST OF FIGURES

Figure 1.1	Velocity and thermal boundary layer.	4
Figure 2.1	Physical model and coordinate system.	23
Figure 2.2	Methodology Flow Chart.	36
Figure 3.1	The velocity profile $f'(\eta)$ with η for various values of magnetic parameter M when $\beta = 0, S = 1, \text{Pr} = 7$ and $\varepsilon = 0.5$.	43
Figure 3.2	The temperature profile $\theta(\eta)$ with η for various values of magnetic parameter M when $\beta = 0, S = 1, \text{Pr} = 7$ and $\varepsilon = 0.5$.	44
Figure 3.3	The velocity profile $f'(\eta)$ with η for various values of magnetic parameter M when $\beta \neq 0, S = 1, \text{Pr} = 7$ and $\varepsilon = 0.5$.	44
Figure 3.4	The temperature profile $\theta(\eta)$ with η for various values of magnetic parameter M when $\beta \neq 0, S = 1, \text{Pr} = 7$ and $\varepsilon = 0.5$.	45
Figure 3.5	The velocity profile $f'(\eta)$ with η for various values of Maxwell parameter β when $\text{Pr} = 7, M = 1, \varepsilon = 0.5$ and $S = 1$.	45
Figure 3.6	The temperature profile $\theta(\eta)$ with η for various values of Maxwell parameter β when $\text{Pr} = 7, M = 1, \varepsilon = 0.5$ and $S = 1$.	46
Figure 3.7	The velocity profile $f'(\eta)$ with η for various values of suction parameter S when $\text{Pr} = 7, M = 1, \varepsilon = 0.5$ and $\beta = 0.8$.	46
Figure 3.8	The temperature profile $\theta(\eta)$ with η for various values of suction parameter S when $\text{Pr} = 7, M = 1, \varepsilon = 0.5$ and $\beta = 0.8$.	47
Figure 3.9	The velocity profile $f'(\eta)$ with η for various values of stretching sheet ε when $\text{Pr} = 7, M = 1, S = 0.5$ and $\beta = 0.8$.	48
Figure 3.10	The temperature profile $\theta(\eta)$ with η for various values of stretching sheet ε when $\text{Pr} = 7, M = 1, S = 0.5$ and $\beta = 0.8$.	48
Figure 3.11	The temperature profile $\theta(\eta)$ with η for various values of Prandtl number Pr when $S = \varepsilon = 0.5, M = 1$ and $\beta = 0.8$	49
Figure 4.1	Physical model and coordinate system.	52

Figure 4.2	The velocity profile $f'(\eta)$ with η for various values of dimensionless velocity slip parameter γ when $\text{Pr} = 7$, $\varepsilon = 0.3$, $\delta = N_R = 0.5$ and $\beta = 0.1$	60
Figure 4.3	The temperature profile $\theta(\eta)$ with η for various values of dimensionless velocity slip parameter γ when $\text{Pr} = 7$, $\varepsilon = 0.3$, $\delta = N_R = 0.5$ and $\beta = 0.1$	61
Figure 4.4	The temperature profile $\theta(\eta)$ with η for various values of thermal slip parameter δ when $\text{Pr} = 7$, $\varepsilon = 0.3$, $\delta = N_R = 0.5$ and $\beta = 0.1$	61
Figure 4.5	The temperature profile $\theta(\eta)$ with η for various values of thermal radiation parameter N_R when $\text{Pr} = 7$, $\varepsilon = \delta = \gamma = 0.5$ and $\beta = 0.1$	62
Figure 4.6	The temperature profile $\theta(\eta)$ with η for various values of Prandtl parameter Pr when $\gamma = \varepsilon = \delta = N_R = 0.5$ and $\beta = 0.1$	63
Figure 4.7	The temperature profile $\theta(\eta)$ with η for various values of Maxwell parameter β when $\varepsilon = \gamma = \delta = N_R = 0.5$ and $\text{Pr} = 7$	64
Figure 4.9	The temperature profile $\theta(\eta)$ with η for various values of stretching parameter ε when $\beta = 0.1$, $\gamma = \delta = N_R = 0.5$ and $\text{Pr} = 7$	65
Figure 5.1	Physical model and coordinate system.	67
Figure 5.2	Variation of $Nu_x \text{Re}_x^{-1/2}$ for various values of $K \geq 0$ when $N_b = N_t = 0.1$, $Le = 15$, $\varepsilon = 0.5$ and $\text{Pr} = 7$	74
Figure 5.3	Variation of $Sh_x \text{Re}_x^{-1/2}$ for various values of $K \geq 0$ when $N_b = N_t = 0.1$, $Le = 15$, $\varepsilon = 0.5$ and $\text{Pr} = 7$	75
Figure 5.4	Variation of $C_f \text{Re}_x^{1/2}$ for various values of $K \geq 0$ when $N_b = N_t = 0.1$, $Le = 15$, $\varepsilon = 0.5$ and $\text{Pr} = 7$	76
Figure 5.5	Variation of $Nu_x \text{Re}_x^{-1/2}$ for various values of $\varepsilon > 0$ when $N_b = N_t = 0.1$, $Le = 15$, $K = 1$ and $\text{Pr} = 7$	77

Figure 5.6 Variation of $Sh_x \text{Re}_x^{-1/2}$ for various values of $\varepsilon > 0$ when
 $N_b = N_t = 0.1, Le = 15, K = 1$ and $Pr = 7$ 78

Figure 5.7 Variation of $C_f \text{Re}_x^{1/2}$ for various values of $\varepsilon > 0$ when
 $N_b = N_t = 0.1, Le = 15, K = 1$ and $Pr = 7$ 78

LIST OF SYMBOLS

a, b	Positive constant for stretching rate
β_0	Induced magnetic field
C	Volume fraction
C_f	Local skin friction coefficient
C_p	Specific heat capacity
C_w	Concentration near the plate
C_∞	Concentration far away from the plate
D	Mass diffusivity
D_B	Brownian diffusion coefficient
D_T	Thermophoresis diffusion coefficient
h_s	Heat transfer coefficient
j_w	Surface mass flux
K	Dimensionless viscoelastic parameter
K_f	Flow consistency parameter
k	Thermal conductivity
k_0	Viscoelastic parameter
k^*	Mean absorption coefficient
Le	Lewis number
M	Magnetic parameter
N_b	Brownian parameter
N_t	Thermophoresis parameter
N_R	Thermal radiation parameter
Nu_x	Local Nusselt number
Pr	Prandtl number
q_r	Radiative heat flux
q_w	Surface heat flux
Re_x	Local Reynolds number

S	Suction parameter
Sh_x	Local Sherwood number
T	Temperature
T_w	Wall temperature
T_∞	Ambient temperature
u, v	Velocity components along x and y axes
$u_w(x)$	Stretching sheet velocity
$u_e(x)$	External velocity

Greek symbols

α_m	Nanofluid thermal diffusivity
β	Maxwell parameter
ε	Stretching parameter
δ	Thermal slip parameter
δ^*	Dimensional thermal slip parameter
γ	Velocity slip parameter
γ^*	Dimensional velocity slip parameter
ρ	Density of nanofluid
ρ^*	Fluid normal stress
σ	Electricity conductivity
σ^*	Stefan Boltzmann constant
ν	Kinematic viscosity
λ	Relaxation time parameter
ψ	Stream function

μ	Dynamic viscosity
τ_w	Shear stress surface
θ	Dimensionless temperature
ϕ	Dimensionless concentration
f	Dimensionless stream function
$(\rho c_p)_f$	Effective heat capacity of nanoparticle

Subscripts

w	Condition at wall
∞	Condition at infinity

LIST OF ABBREVIATIONS

IVP	Initial Value Problem
MHD	Magnetohydrodynamic
PWT	Prescribed Wall Temperature
RKF	Runge-Kutta Fehlberg

REFERENCES

- Abel, M.S., Tawade, J.V., and Nandeppanavar, M. M. (2012). MHD flow and heat transfer for the upper-convected Maxwell fluid over a stretching sheet. *Meccanica*, 47, 385–393.
- Acheson, D. J. (1990). Elementary fluid dynamics. In *Oxford University Press*.
- Afify, A. A. (2009). Similarity solution in MHD: Effects of thermal diffusion and diffusion thermo on free convective heat and mass transfer over a stretching surface considering suction or injection. *Communications in Nonlinear Science and Numerical Simulation*, 14(5), 2202–2214.
- Ahmad, S., and Pop, I. (2010). Mixed convection boundary layer flow from a vertical flat plate embedded in a porous medium filled with nanofluids. *International Communications in Heat and Mass Transfer*, 37, 987–991.
- Ahmad, S. B. (2009). *Convection boundary layer flows over needles and cylinders in viscous fluids*. Ph.D Thesis, Universiti Putra Malaysia, Malaysia.
- Ali, F. M., Nazar, R., and Ariffin, N. M. (2013). Effect of thermal radiation on unsteady stagnation-point flow with mass transfer. *AIP Proceeding of Simposium Kebangsaan Sains Matematik Ke-20 (SKSM20)*, 1522, 80–85.
- Ali, F. M., Nazar, R., Ariffin, N. M., and Pop, I. (2011). Effect of Hall current on MHD mixed convection boundary layer flow over a stretched vertical flat plate. *Meccanica*, 46(5), 1103–1112.
- Ali, M. E. (1994). Heat transfer characteristics of a continuous stretching surface. *Warme Stoffubertragung*, 29, 227–234.
- Ali, M. E. (1995). On thermal boundary layer on a power-law stretched surface with suction or injection. *International Journal of Heat and Fluid Flow*, 16, 280–290.
- Aly, E. H. (2015). Effect of the velocity slip boundary condition on the flow and heat transfer of nanofluid over a stretching sheet. *Journal of Computational and Theoretical Nanoscience*, 12(9), 2428–2436.
- Ariel, P. T. H. and S. A. (2006). The flow of an elastico-viscous fluid past a stretching sheet with partial slip. *Acta Mechanica*, 187(1–4), 29–35.

- Bachok, N., Ishak, A., and Pop, I. (2011). Stagnation-point flow over a stretching/shrinking sheet in a nanofluid. *Nanoscale Research Letters*, 6(1), 623.
- Bachok, N., Ishak, A., and Pop, I. (2012). Boundary layer stagnation-point flow and heat transfer over an exponentially stretching/shrinking sheet in a nanofluid. *International Journal of Heat and Mass Transfer*, 55(25), 8122–8128.
- Baehr, H. D., and Stephan, K. (2006). Heat and Mass Transfer. *Berlin Heidelberg: Springer Verlag*.
- Bakier, A.Y., and Gorla, R. S. R. (1996). Thermal radiation effect on mixed convection from horizontal surfaces in saturated porous media. *Transport Porous Media*, 23, 357–363.
- Bataller, R. C. (2008). Radiation effects for the Blasius and Sakiadis flows with convective surface boundary condition. *Applied Mathematics and Computation*, 206(2), 832–840.
- Bejan, A. (2013). *Convection heat transfer*. (J. W. & Sons, Ed.).
- Bhattacharyya, S. N., and Gupta, A. S. (1985). On the linear stability of viscous flow over a stretching sheet. *Q. Appl. Math.*, 43, 359–367.
- Bhattacharyya, K. (2011). Effects of radiation and heat source/sink on unsteady MHD boundary layer flow and heat transfer over a shrinking sheet with suction/injection. *Frontiers of Chemical Science and Engineering*, 5(3), 376–384.
- Bhattacharyya, K., and Layek, G. C. (2011). Effects of suction/blowing on steady boundary layer stagnation-point flow and heat transfer towards a shrinking sheet with thermal radiation. *International Journal of Heat and Mass Transfer*, 54(1–3), 302–307.
- Bhatti, M.M., and Rashidi, M. M. (2016). Effects of thermo-diffusion and thermal radiation on Williamson nanofluid over a porous shrinking/ stretching sheet. *Journal of Molecular Liquids*, 221, 567–573.
- Bird, R. B., Armstrong, R. C., and Hassager, O. (1987). *Dynamics of polymeric liquids* (Volume 1). Wiley, New York.

- Buongiorno, J. (2006). Convective transport in nanofluids. *ASME Journal of Heat Transfer*, 128(3), 240–250.
- Burmeister, L. C. (1993). Convective Heat transfer. In *New York: John Wiley and Sons Inc.*
- Carragher, P., and Crane, L. J. (1982). Heat Transfer on a Continuous Stretching Sheet. *Z. Angew. Math. Mech.*, 62, 564–565.
- Cebeci, T., and Bradshaw, P. (2012). Physical and computational aspects of convective heat transfer. *Springer Science & Business Media.*
- Chakrabarti, A., and Gupta, A. S. (1979). Hydromagnetic flow and heat transfer over a stretching sheet. *Quart. Appl. Math.*, 37, 73–78.
- Chandrasekar, M., and Suresh, S. (2009). A review of the mechanisms of heat transport in nanofluids. *Heat Transfer Engineering*, 30, 1136–1150.
- Chen, C. K., and Char, M. (1988). Heat transfer of a continuous, stretching surface with suction or blowing. *Journal of Mathematical Analysis and Applications*, 135, 568–580.
- Chiam, T. C. (1995). Hydromagnetic flow over a surface stretching with a power-law velocity. *Int. J. Engng Sci.*, 33, 429–435.
- Choi, S. (1995). Enhancing thermal conductivity of fluids with nanoparticles in developments and applications of non-Newtonian flows. In: *D.A. Siginer, H.P. Wang, Editors, ASME*, 66, 99–105.
- Choi, S. U. S. (1995). Enhancing thermal conductivity of fluids with nanoparticles. *ASME International Mechanical Engineering Congress and Exposition*, 231, 99–105.
- Cortell, R. (2007). Viscous flow and heat transfer over a nonlinearly stretching sheet. *Applied Mathematics and Computation*, 184, 864–873.
- Cortell, R. (2008). Effects of viscous dissipation and radiation on the thermal boundary layer over a nonlinearly stretching sheet. *Physics Letters A*, 372, 631–636.
- Crane, L. J. (1970). Flow past a stretching plate. *Z. Angew. Math. Phys.*, 21, 645–647.

- Dandapat, B. S., Holmedal, L. E., and Andersson, H. I. (1994). Stability of flow of a viscoelastic fluid over a stretching sheet. *Arch. Mech.*, 46, 829–838.
- Das, S. K., Choi, S.U., and Patel, H. E. (2006). Heat transfer in nanofluids a review. *Heat Transfer Engineering*, 27(10), 3–19.
- Devi, S. P. A., and Thiagarajan, M. (2006). Steady nonlinear hydromagnetic flow and heat transfer over a stretching surface of variable temperature. *Heat and Mass Transfer*, 42(8), 671–677.
- Eastman, J. A., Choi, U.S., Li, S., Thompson, L.J., and Lee, S. (1996). Enhanced thermal conductivity through the development of nanofluids. In *In MRS proceedings* (Vol. 457, p. 3). Cambridge University Press.
- Elbashbeshy, E. M. A. (1998). Heat transfer over a stretching surface with variable surface heat flux. *J. Phys. D: Appl. Phys.*, 31, 1951–1954.
- Ferdows, M., Khan, M. S., Alam, M. M., and Sun, S. (2012). MHD mixed convective boundary layer flow of a nanofluid through a porous medium due to an exponentially stretching sheet. *Mathematical Problems in Engineering*, 2012, 1–21.
- Gad-el-Hak, M. (1999). The fluid mechanics of microdevices-the Freeman scholar lecture. *Transactions-American Society of Mechanical Engineers Journal of FLUIDS Engineering*, 121, 5–33.
- Gireesha, B. J., Gorla, R. S. R., and Mahanthesh, B. (2015). Effect of suspended nanoparticles on three-dimensional MHD flow, heat and mass transfer of radiating Eyring-Powell fluid over a stretching sheet. *Journal of Nanofluids*, 4(4), 474–484.
- Gorla, R. S. R., and Sidawi, I. (1994). Free convection on a vertical stretching surface with suction and blowing. *Appl. Sci. Res.*, 52, 247–257.
- Gorla, R.S.R., and Gireesha, B. J. (2016). Dual solutions for stagnation-point flow and convective heat transfer of a Williamson nanofluid past a stretching/shrinking sheet. *Heat and Mass Transfer*, 52(6), 1153–1162.
- Gupta, P. S., and Gupta, A. S. (1977). Heat and mass transfer on a stretching sheet with suction or blowing. *Can. J. Chem. Eng.*, (55), 744–746.
- Ha, S. (2001). A nonlinear shooting method for two-point boundary value problems. *Computers & Mathematics with Applications*, 42(10), 1411–1420.

- Hamad, M. A. A., Pop, I., and Ismail, A. I. M. (2011). Magnetic field effects on free convection flow of a nanofluid past vertical semi-infinite flat plate. *Nonlinear Analysis: Real World Applications*, 12, 1338–1346.
- Haq, R., Nadeem, S., Khan, Z., and Okedayo, T. (2014). Convective heat transfer and MHD effects on Casson nanofluid flow over a shrinking sheet. *Open Physics*, 12(12), 862–871.
- Hassanien, I. A., Abdullah, A. A., and Gorla, R. S. R. (1998). Flow and Heat Transfer in a Power-Law Fluid over a Nonisothermal Stretching Sheet. *Math. Comput. Modell.*, 28, 105.
- Hayat, T., Abbas, Z., and Sajid, M. (2006). Series solution for the upper-convected Maxwell fluid over a porous stretching plate. *Physics Letters, Section A: General, Atomic and Solid State Physics*, 358(5–6), 396–403.
- Hayat, T., Waqas, M., Shehzad, S. A., and Alsaedi, A. (2016). Chemically reactive flow of third grade fluid by an exponentially convected stretching sheet. *Journal of Molecular Liquids*, 223, 853–860.
- Hiemenz, K. (1911). Boundary layer for a homogeneous flow around a dropping cylinder. *Dinglers Polytech. J.*, 326, 321–410.
- Ibrahim, W., and Shankar, B. (2013). MHD boundary layer flow and heat transfer of a nanofluid past a permeable stretching sheet with velocity, thermal and solutal slip boundary conditions. *Computers and Fluids*, 75, 1–10.
- Imtiaz, M., Hayat, T., Alsaedi, A., and Hobiny, A. (2016). Homogeneous-heterogeneous reactions in MHD flow due to an unsteady curved stretching surface. *Journal of Molecular Liquids*, 221, 245–253.
- Ishak, A., Nazar, R., and Pop, I. (2007). Mixed convection on the stagnation-point flow toward a vertical, continuously stretching sheet. *J. Heat Transfer*, 129, 1087–1090.
- Jang, S. P., and Choi, S. U. (2004). Role of Brownian motion in the enhanced thermal conductivity of nanofluids. *Applied Physics Letters*, 84(21), 4316–4318.
- Javed, T., Abbas, Z., Sajid, M., and Ali, N. (2011). Heat transfer analysis for a hydromagnetic viscous fluid over a non-linear shrinking sheet. *International Journal of Heat and Mass Transfer*, 54(9–10), 2034–2042.

- Jusoh, R., Nazar, R., and Pop, I. (2017). Flow and heat transfer of magnetohydrodynamic three-dimensional Maxwell nanofluid over a permeable stretching/shrinking surface with convective boundary conditions. *International Journal of Mechanical Sciences*, 124–125, 166–173.
- Kameswaran, P. K., Sibanda, P., RamReddy, C., and Murthy, P. V. S. N. (2013). Dual solutions of stagnation-point flow of a nanofluid over a stretching surface. *Boundary Value Problems*, 2013(1), 188.
- Kandasamy, R., Muhaimin, I., and Saim, H. B. (2010). Lie group analysis for the effect of temperature-dependent fluid viscosity with thermophoresis and chemical reaction on MHD free convective heat and mass transfer over a porous stretching surface in the presence of heat source/sink. *Communications in Nonlinear Science and Numerical Simulation*, 15(8), 2109–2123.
- Khan, W. A., and Pop, I. (2010). Boundary-layer flow of a nanofluid past a stretching sheet. *International Journal of Heat and Mass Transfer*, 53(11–12), 2477–2483.
- Kuznetsov, A.V., and Nield, D. A. (2010). Natural convective boundary-layer flow of a nanofluid past a vertical plate. *International Journal of Thermal Sciences*, 49, 243–247.
- Lapwood, E. R. (1948). Convection of a fluid in a porous medium. *In Mathematical Proceedings of the Cambridge*, 44(4), 508–521.
- Latorre, M., and Rinaldi, C. (2009). Applications of magnetic nanoparticles in medicine: magnetic fluid hyperthermia. *Puerto Rico Health Science Journal*, 28, 227–238.
- Lienhard IV, J. H., and Lienhard V, J. H. (2006). *A Heat Transfer Textbook*. Cambridge: Phlogiston Press.
- Liu, I. C. (2005). A note on heat and mass transfer for a hydromagnetic flow over a stretching sheet. *International Communications in Heat and Mass Transfer*, 32(8), 1075–1084.
- Lok, Y. Y., Ishak, A. M., and Pop, I. (2011). MHD stagnation-point flow towards a shrinking sheet. *International Journal of Numerical Methods for Heat & Fluid Flow*, 21(1), 61–72.

- Madhu, M., and Kishan, N. (2016). Finite element analysis of heat and mass transfer by MHD mixed convection stagnation-point flow of a non-Newtonian power-law nanofluid towards a stretching surface with radiation. *Journal of the Egyptian Mathematical Society*, 24(3), 458–470.
- Mahmoudi, A., Mejri, I., Abbassi, M. A., and Omri, A. (2015). Analysis of MHD natural convection in a nanofluid-filled open cavity with non uniform boundary condition in the presence of uniform heat generation/absorption. *Powder Technology*, 269, 275–289.
- Majumder, M., Chopra, N., Andrews, R., and Hinds, B. J. (2005). Nanoscale hydrodynamics: enhanced flow in carbon nanotubes. *Nature*, 438(7064), 44–48.
- Mahapatra, T. R., and Gupta, A. S. (2002). Heat transfer in stagnation-point flow towards a stretching sheet. *Heat and Mass Transfer*, 38(6), 517–521.
- Makinde, O. D., and Aziz, A. (2011). Boundary layer flow of a nanofluid past a stretching sheet with a convective boundary condition. *International Journal of Thermal Sciences*, 50(7), 1326–1332.
- Makinde, O. D., Reddy, M. G., and Reddy, K. V. (2017). Effects of thermal radiation on MHD peristaltic motion of Walters-B fluid with heat source and slip conditions. *Journal of Applied Fluid Mechanics*, 10(4), 1105–1112.
- Malik, M. Y., Naseer, M., Nadeem, S., and Rehman, A. (2014). The boundary layer flow of Casson nanofluid over a vertical exponentially stretching cylinder. *Applied Nanoscience*, 4(7), 869–873.
- Mamaloukas, C., Abel, M. S., Tawade, J. V., and Mahabaleswar, U. S. (2011). On effects of a transverse magnetic field on an UCM fluid over a stretching sheet. *International Journal of Pure and Applied Mathematics*, 66(1), 1–9.
- Maxwell, J. (1867). On the dynamical theory of gases. *Philosophical Transactions*, 96, 49–88.
- Merkin, J. H. (1994). Natural-convection boundary-layer flow on a vertical surface with Newtonian heating. *International Journal of Heat and Fluid Flow*, 15(5), 392–398.
- Mohamed, M. K. A., Noor, N. A. Z., and Salleh, M. Z. (2015). Stagnation point flow past a stretching sheet in a nanofluid with slip condition. In *AIP Proc. Int. Stat. Conf.*, 1645, 635.

- Mohamed, M. K. A. Noar, N. A. Z., Salleh, M. Z., and Ishak, A. (2017). Slip flow on Stagnation Point over a Stretching Sheet in a Viscoelastic Nanofluid. In *The Journal of Chemical and Physics*, 1830, 020015.
- Mohamed M. K. A., Salleh M. Z., Nazar, R., and Ishak, A. (2013). Numerical investigation of stagnation point flow over a stretching sheet with convective boundary conditions. *Boundary Value Problems*, 4, 1–10.
- Mohamed M. K. A., Salleh M. Z., Ishak, A., and Pop, I. (2015). Stagnation point flow and heat transfer over a stretching/shrinking sheet in a viscoelastic fluid with convective boundary condition and partial slip velocity. *The European Physical Journal Plus*, 130(8), 171.
- Mohammadein, A. A., and El-Amin, M. F. (2000). Thermal dispersion-radiation effects on non-Darcy natural convection in a fluid saturated porous medium. *Transport Porous Medium*, 40(2), 153–163.
- Morrison, D. D, Riley, J. D., and Zancanaro, J. F. (1962). Multiple shooting method for two-point boundary value problems. *Communications of the ACM*, 5(12), 613–614.
- Muhaimin, Kandasamy, R., Khamis, A. B. (2008). Effects of heat and mass transfer on nonlinear MHD boundary layer flow over a shrinking sheet in the presence of suction. *Applied Mathematics and Mechanics*, 29 (10), 1309–1317.
- Mustafa, M., Hayat, T., Pop, I., Asghar, S., and Obaidat, S. (2011). Stagnation-point flow of a nanofluid towards a stretching sheet. *International Journal of Heat and Mass Transfer*, 54, 5588–5594.
- Nadeem, S., Haq, R. U., and Akbar, N. S. (2014). MHD three-dimensional boundary layer flow of Casson nanofluid past a linearly stretching sheet with convective boundary condition. *IEE E Transactions on Nanotechnology*, 13(1), 109–115.
- Nadeem, S., and Hussain, S. T. (2014). Flow and heat transfer analysis of Williamson nanofluid. *Applied Nanoscience*, 4(8), 1005–1012.
- Nadeem, S., U1 Haq, R., and Khan, Z. H. (2014). Heat transfer analysis of water-based nanofluid over an exponentially stretching sheet. *Alexandria Engineering Journal*, 53(1), 219–224.

- Noghrehabadi, A., Pourrajab, R., and Ghalambaz, M. (2012). Effect of partial slip boundary condition on the flow and heat transfer to nanofluids past stretching sheet prescribed constant wall temperature. *International Journal of Thermal Sciences*, 54, 253–261.
- Noghrehabadi, A., Pourrajab, R., and Ghalambaz, M. (2013). Flow and heat transfer of nanofluids over stretching sheet taking into account partial slip and thermal convective boundary conditions. *Heat and Mass Transfer*, 49, 1357–1366.
- Oahimire, J. I., and Olajuwon, B. I. (2014). Effect of Hall current and thermal radiation on heat and mass transfer of a chemically reacting MHD flow of a micropolar fluid through a porous medium. *Journal of King Saud University - Engineering Sciences*, 26(2), 112–121.
- Oldroyd, J. G. (1950). On the formulation of rheological equations of state. In *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* 200(1063), 523–541.
- Özisik. (1985). *Heat transfer: a basic approach*. McGraw-Hill College.
- Pal, D., Mandal, G., and Vajravelu, K. (2013). MHD convection–dissipation heat transfer over a non-linear stretching and shrinking sheets in nanofluids with thermal radiation. *International Journal of Heat and Mass Transfer*, 65, 481–490.
- Pal, D., and Mondal, H. (2011). MHD non-Darcian mixed convection heat and mass transfer over a non-linear stretching sheet with Soret-Dufour effects and chemical reaction. *International Communications in Heat and Mass Transfer*, 38(4), 463–467.
- Pahlavan, A. A., Aliakbar, V., Farahani, F. V., and Sadeghy, K. (2009). MHD flows of UCM fluids above porous stretching sheets using two-auxiliary-parameter homotopy analysis method. *Communications in Nonlinear Science and Numerical Simulation*, 14(2), 473–488.
- Pohlhausen, E. (1921). Der Wärmeaustausch zwischen festen Körpern und Flüssigkeiten mit kleiner Reibung und kleiner Wärmeleitung. *Journal of Applied Mathematics and Mechanics (ZAMM)*, 1, 115–121.
- Prabhavathi, B., Sudarsana Reddy, P., & Bhuvana Vijaya, R. (2018). Heat and mass transfer enhancement of SWCNTs and MWCNTs based Maxwell nanofluid flow over a vertical cone with slip effects. *Powder Technology*, 340, 253–263.

- Prasannakumara, B.C., Gireesha, B.J., Gorla, R.S. and Krishnamurthy, M. R. (2016). Effects of chemical reaction and nonlinear thermal radiation on Williamson nanofluid slip flow over a stretching sheet embedded in porous medium. *Journal of Aerospace Engineering*, 29(5), 04016019.
- Raisi, A., Ghasemi, B., and Aminossadati, S. M. (2011). A numerical study on the forced convection of laminar nanofluid in a microchannel with both slip and no slip conditions. *Numerical Heat Transfer, Part A: Applications*, 59(2), 114–129.
- Rakesh, K., and Khem, C. (2011). Effect of slip condition and hall current on unsteady MHD flow of a viscoelastic fluid past an infinite vertical porous plate through porous medium. *Int. J. Eng. Sci. Technol.*, 3(4), 0975–5462.
- Sadeghy, K., Hajibeygi, H., and Taghavi, S. M. (2006). Stagnation-point flow of upper-convected Maxwell fluids. *International Journal of Non-Linear Mechanics*, 41(10), 1242–1247.
- Saffman, P.G., and Taylor, G. (1958). The penetration of a fluid into a porous medium or Hele-Shaw cell containing a more viscous liquid. *In Proceedings of the Royal Society of London A: Mathematical, Physical and Engineering Sciences*, 245(1242), 312–329.
- Sakiadis, B. C. (1961). Boundary-layer behaviour on continuous solid surfaces: I. Boundary layer equations for two-dimensional and axisymmetric flow. *Am. Inst. Chem. Engineers J.*, 26–28.
- Salloum, M., Ma, R., Weeks, D., and Zhu, L. (2008). Controlling nanoparticle delivery in magnetic nanoparticle hyperthermia for cancer treatment: experimental study in agarose gel. *International Journal of Hyperthermia*, 24, 337–345.
- Sarpkaya, T. (1961). Flow of non-Newtonian fluids in a magnetic field. *AIChE J.*, 7, 324–328.
- Schlichting, H. (1979). *Boundary-layer theory* (Vol. 7). New York: McGraw-Hill.
- Selimefendigil, F., and Öztop, H. F. (2014). Numerical study of MHD mixed convection in a nanofluid filled lid driven square enclosure with a rotating cylinder. *International Journal of Heat and Mass Transfer*, 78, 741–754.
- Shima, P. D., Philip, J., and Raj, B. (2009). Magnetically controllable nanofluid with tunable thermal conductivity and viscosity. *Applied Physics Letters*, 95(13), 133112.

- Sivashanmugam, P. (2012). Application of nanofluids in heat transfer. *INTECH Open Access Publisher*.
- Suali, M., Nik Long, N. M. A., and Ariffin, N. M. (2012). Unsteady Stagnation Point Flow and Heat Transfer over a Stretching/Shrinking Sheet with Suction or Injection. *Journal of Applied Mathematics*, 2012, 12
- Sulochana, C., and Sandeep, N. (2015). Dual solutions for radiative MHD forced convective flow of a nanofluid over a slendering stretching sheet in porous medium. *Applied Nanoscience*, 6(3), 451–459.
- T. R. Mahapatra and A. S. Gupta. (2001). MHD stagnation point flow towards a stretching sheet. *Acta Mechanica*, 152, 191–196.
- Takhar, H. S., All, M. A., and Gupta, A. S. (1989). Stability of magnetohydrodynamic flow over a stretching sheet. In: *Liquid Metal Hydrodynamics (Lielpeteris J., Moreau, R., Eds.)*, 465–471.
- Taylor, R., Coulombe, S., Otanicar, T., Phelan, P., Gunawan, A., Lv, W., Rosengarten, G., Prasher, R., and Tyagi, H. (2013). Small particles, big impacts: a review of diverse applications of nanofluids. *Journal of Applied Physics*, 113(1), 1.
- Tsou, F., Sparrow, E., and Goldstein, R. (1967). Flow and heat transfer in the boundary layer on a continuous moving surfaces. *Int. J. Heat Mass Transfer*, 10, 219–235.
- Turkyilmazoglu, M. (2011). Multiple solutions of heat and mass transfer of MHD slip flow for the viscoelastic fluid over a stretching sheet. *International Journal of Thermal Sciences*, 50(11), 2264–2276.
- Vafai, K. (1984). Convective flow and heat transfer variable-porosity media. *Journal of Fluid Mechanics*, 147, 233–259.
- Wang, C. Y. (1989). Free convection on a vertical stretching surface. *J. Appl. Math. Mech. (ZAMM)*, 69, 418–420.
- Wang, C. Y. (2008). Stagnation flow towards a shrinking sheet. *International Journal of Non-Linear Mechanics*, 43(5), 377–382.
- Wong, S. W., Awang, M. O., and Ishak, A. (2011). Stagnation-Point Flow over an Exponentially Shrinking/Stretching Sheet. *Zeitschrift Fur Naturforschung A- Journal of Physical Sciences*, 66(12), 705.

- Yang, F. (2009). Slip boundary condition for viscous flow over solid surfaces. *Chemical Engineering Communications*, 197(4), 544–550.
- Yoshimura, A., and Prud'homme, R. K. (1988). Wall slip correction. *Journal of Rheology*, 32(1), 53–67.